What you'll Learn About

- Definition of the derivative
- Notation

$$\boxed{\lim_{x \to a} \frac{f(x)-f(a)}{x-a}}$$

$$\boxed{\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$$

Use the substitution h = x - a to create ~~the~~ another definition of the derivative

$\Delta x = h$

A₁) Set-up a formula for the slope of f(x) = x^2 at x = -1

$$\boxed{\lim_{x \to -1} \frac{x^2-1}{x+1}} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} = -2$$

$$\lim_{\substack{h-1 \to -1 \\ +1 \quad +1}} \frac{(h-1)^2-1}{h} = \boxed{\lim_{h \to 0} \frac{(h-1)^2-1}{h}}$$

$$\frac{h^2-2h+1-1}{h} \qquad \frac{\cancel{h}(h-2)}{\cancel{h}}$$

A₂) Use the substitution h = x - a to set-up the definition of the derivative

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a} \qquad \boxed{\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$$

$$\frac{1}{x-2} = f(x)$$

$$\frac{1}{(5)-2} = f(5)$$

$$\frac{1}{(x+h)-2} = f(x+h)$$

B₁) Set-up a formula for the slope of $f(x) = \frac{1}{x-2}$ at $x = 4$

$$\boxed{\lim_{x \to 4} \frac{\frac{1}{x-2} - \frac{1}{2}}{x-4}}$$

$$\lim_{h \to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h}$$

$$\boxed{\lim_{h \to 0} \frac{\frac{1}{4+h-2} - \frac{1}{4-2}}{h}}$$

Δx

B₂) Use the substitution h = x - a to set-up the definition of the derivative

$h = x - 4$

$\boxed{h+4 = x}$

$$\lim_{h+4 \to 4} \frac{\frac{1}{(h+4)-2} - \frac{1}{2}}{h+4-4}$$

$$\boxed{\lim_{h \to 0} \frac{\frac{1}{h+2} - \frac{1}{2}}{h}}$$

Given a definition of the derivative(slope) find the function that you are taking the derivative of and the point you are finding the derivative(slope) at

$\lim_{x \to a} \frac{f(x)-f(a)}{x-4}$

A) $\boxed{\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}}$

$f(x) = \sqrt{x}$

$x = 4 \quad y = 2$

B) $\lim_{x \to 2} \frac{\ln x - \ln 2}{x-2}$

$f(x)$; $f(a)$; a

$f(x) = \ln x \qquad (2, \ln 2)$

$\lim_{h \to} \frac{f(x+h)-f(x)}{h}$

C) $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

$f(x+h)$; $f(x) \to f(2)$

$f(x) = x^3 \qquad (2, 8)$

D) $\lim_{h \to 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h} \to f(x) = \frac{2}{x} \qquad \left(3, \frac{2}{3}\right)$

Another Definition: $\lim_{h \to 0} \frac{f(x+h)-f(x-h)}{2h}$